# Mode conversion in periodically disturbed thin-film waveguides*

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Mode conversion in a periodically perturbed thin-film optical waveguide is studied in detail. Three different types of perturbations are considered: periodic index of refraction of the film, periodic index of refraction of the substrate, and periodic boundary. The applications in filters, mode converters, and distributed feedback lasers are discussed.

## I. INTRODUCTION

Periodic structures have a large field of applications in active and passive optical thin-film structures. Their stop-band passband characteristic can be used for distributed feedback, [1–5] filtering, [4,5] and coupling.[6] Their space-harmonic characteristic can be mainly used for phase-matched nonlinear interactions[7,8] and for coupling to drifting electrons in a thin-film semiconductor.

In a recent paper,[4] the authors have studied the coupling between identical modes in different types of periodic optical thin-film waveguides (surface periodicity, waveguide index periodicity, and substrate periodicity). In this paper we extend our previous work to the case of coupling between nonidentical modes and we evaluate the efficiency of the mode conversion and its use for mode generation, filtering, and distributed feedback. The last application consisting of the use of a higher-order mode to carry the feedback function in a DFB (distributed feedback) laser operating on a lower forward mode. As we will show in this paper, this scheme is sometimes more efficient than in the case where the feedback and direct waves are in the same mode. We also show that in a number of cases, the coupling coefficient (for mode conversion, feedback, or filtering) reaches a maximum in a specific frequency region thus allowing optimization of the system.

The approach used in this paper is slightly different than in Ref. 4; however, for the sake of continuity and to avoid repetitions, our previous notations and a number of results and figures will be used here. Our study is limited to the case of TE waves, but the approach can be applied in a straightforward manner, for TM waves.

We shall study three important types of periodic structures [Fig. 2 upper right-hand corner, or Figs. 1(a) and 1(b) Ref. 4]: periodically inhomogeneous thin-film guides, periodically inhomogeneous substrate guide, and thin-film waveguide with period surfaces. All these structures are technologically feasible.[1–3]

## II. WAVE SOLUTION AND BRILLOUIN DIAGRAM

Electromagnetic waves can be guided in a structure which consists of a thin-film dielectric of relative permittivity $\epsilon_1$ imbedded in a medium of refractive permittivity $\epsilon_2 < \epsilon_1$. For a transverse electric (TE) guided wave the field expression is[4,9]

$$\mathbf{E} = \mathbf{e}_y C u(x) \exp(i\kappa z), \tag{1a}$$

$$\begin{aligned} u(x) &= \cos(sx)/\cos(sL) && (\text{even modes}, \ |x| \le L) \\ &= \sin(sx)/\sin(sL) && (\text{odd modes}, \ |x| \le L) \\ &= \exp(\delta L - \delta|x|) && (\text{even modes}, \ |x| \ge L) \\ &= \operatorname{sign}(x) \exp(\delta L - \delta|x|) && (\text{odd modes}, \ |x| \ge L), \end{aligned} \tag{1b}$$

where $2L$ is the waveguide thickness, $C$ is the field at the the boundary, and $s$, $\delta$, and $\kappa$ are the components of the wave vector. These wave vectors are related to the frequency $\omega/2\pi$ by the disperson relations

$$\kappa^2 + s^2 = \epsilon_1 k^2, \tag{2a}$$

$$\kappa^2 - \delta^2 = \epsilon_2 k^2, \tag{2b}$$

$$\begin{aligned} \delta &= s \ \tan(sL) && (\text{even modes}) \\ &= s[-\cot\text{an}(sL)] && (\text{odd modes}), \end{aligned} \tag{2c}$$

where $k = \omega/c$. The above relations have multiple solutions which correspond to the different modes.

If the optical guide is periodic, the field would consist of an infinite number of space harmonics[10] of longitudinal wave vectors

$$\kappa_{pn} = \kappa_p + nK, \tag{3}$$

where $p$ is the mode index, $n$ is the space-harmonic index, $K = 2\pi/\Lambda$ where $\Lambda$ is the perturbation period, and $\kappa_p$ is to be determined. The corresponding Brillouin diagram consists of an infinite number of subdiagrams, each identical to the diagram of a homogeneous thin-film waveguide (Fig. 2, Ref. 4). Strong phase-matched coupling occurs at the intersection points between different harmonics, leading to reflection or mode conversion. Two types of coupling could occur.

(i) Codirectional where the group velocities of the two coupled harmonics are parallel [Fig. 3(a)]. In this case the energy is transferred back and forth between the two harmonics.

(ii) Contradirectional, where the group velocities are antiparallel [Fig. 3(b)]. In this case, there is a one-way energy transfer.

## III. MODE CONVERSION IN A PERIODICALLY INHOMOGENEOUS GUIDE

Let us consider the case of a periodically inhomogeneous guide imbedded in a homogeneous substrate, where

$$\epsilon_1(z) = \epsilon_1[1 + \eta \cos(Kz)], \quad \text{with } \eta \ll 1 \tag{4}$$

$$\epsilon_2 = \text{const}$$

Without any loss of generality, we consider the interaction between the $n=0$ space harmonic of the $p$th mode and the neighboring $n=\pm 1$ space harmonics of the $q$th mode. The phase-matching condition is

$$\kappa_p \pm \kappa_q = K. \tag{5}$$

The plus and minus signs correspond, respectively, to the contradirectional and codirectional interaction. Let $\omega_{pq}$ be the frequency at which this condition is satisfied. For $\eta$ small, all the other space harmonics can be neglected.

To understand and formulate the coupling mechanism, let us consider the $p$th mode wave of frequency $\omega = \omega_{pq} + \Delta\omega$. The corresponding electric field can be written

$$E_{po} = C_{po} u'_{po}(x) \exp(i\kappa'_p z), \tag{6}$$

where $\kappa'_p = \kappa_p + \Delta\kappa$, and $u'_{po}(x)$ is given by Eq. 1(b) with $s$ and $\delta$ replaced by $s'_p = s_p + \Delta s_p$ and $\delta'_p = \delta_p + \Delta\delta_p$, respectively. The terms $\kappa_p$, $s_p$, and $\delta_p$ correspond to the homogeneous guide, and $\Delta\kappa$, $\Delta s_p$, and $\Delta\delta_p$ are small perturbations caused by the inhomogenety. Due to the presence of the periodic component in the dielectric constant [Eq. (3)] a spatially periodic convection current $J_c$ is generated:

$$\begin{aligned} J_c &= -i\omega\eta\epsilon_0\epsilon_1 \cos(Kz) E_{po} \\ &= -i\omega\eta \tfrac{1}{2}(\epsilon_0\epsilon_1) C_{po} u'_{po}(x) h(x) \\ &\quad \times \{\exp[i(\kappa'_p + K)z] + \exp[i(\kappa'_p - K)z]\} \end{aligned} \tag{7}$$

where $h(x)=1$ for $|x|<L$ and $h(x)=0$ for $|x|>L$. This current will excite the two neighboring space harmonics. Let us consider the case of contradirectional longitudinal phase matching. Then a backward $q$th mode wave $E_{q,-1}$ will be excited. However to determine the effective excitation current for the new $q$th mode, the current $J_c$ has to be expanded as a function of the transverse modes:

$$u'_{po}(x) h(x) = \textstyle\sum_j a'_{pj} u_j(x) \tag{8}$$

and only the term $a'_{pj}$ has to be taken into account. This corresponds to transverse phase matching. The expansion coefficients $a'_{pj}$ are given by

$$\begin{aligned} a'_{pj} &= \left[\int_{-\infty}^{+\infty} u'_p(x) h(x) u_j^*(x)\,dx\right]\left[\int_{-\infty}^{+\infty} u_j(x) u_j^*(x)\,dx\right]^{-1} \\ &= \left[\int_{-L}^{+L} u'_p(x) u_j^*(x)\,dx\right]\left[\int_{-\infty}^{+\infty} u_j(x) u_j^*(x)\,dx\right]. \end{aligned} \tag{9}$$

Then the corresponding wave equation is

$$\begin{aligned} &\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} + \epsilon_1 \frac{\omega^2}{c^2}\right) E_{q,-1} \\ &\quad = -\eta \frac{\omega^2}{2} \frac{\epsilon_1}{c} C_{po} a'_{pq} u_q(x) \exp[i(\kappa'_p - K)z] \end{aligned} \tag{10}$$

for $|x|<L$, and for $|x|>L$ we replace $\epsilon_1$ by $\epsilon_2$ in the parentheses. Replacing $E_{q,-1}$ by an expression similar to Eq. (6), we obtain

$$\begin{aligned} &[(s_q + \Delta s_q)^2 + (\kappa_q + \Delta\kappa)^2 - \epsilon_1 \omega^2/c^2] C_{q,-1} u'_q(x) \\ &\quad = \eta \frac{\omega^2}{2} \frac{\epsilon_1}{c^2} C_{po} a'_{pq} u_q(x) \exp[i(\kappa'_p + \kappa'_q - K)z] \end{aligned} \tag{11}$$

for $|x|<L$ and

$$\begin{aligned} &[-(\delta_q + \Delta\delta_q)^2 + (\kappa_q + \Delta\kappa)^2 - \epsilon_2 \omega^2/c^2] C_{q,-1} u'_q(x) \\ &\quad = \eta \frac{\omega^2}{2} \frac{\epsilon_1}{c^2} C_{po} a'_{pq} u_q(x) \exp[i(\kappa'_p + \kappa'_q - K)z] \end{aligned} \tag{12}$$

for $|x|>L$. Using the dispersion relation for the unperturbed case [Eq. (2)], neglecting second-order terms (i.e., small term $\times u'_q =$ small term $\times u_q$), and expressing $\Delta s_q$ and $\Delta\delta_q$ as function of $\Delta\kappa_q$ and $\Delta\omega$ by differentiating Eqs. (2a)–(2c). Equations (12) and (13) reduce to

$$\left(-\frac{\Delta\kappa}{\kappa_q} + B_q \frac{\Delta\omega}{\omega_{pq}}\right) C_{q,-1} = \eta \frac{\epsilon_1}{4} \left(\frac{\omega}{c\kappa_q}\right)^2 a_{pq} C_{p,0}, \tag{13}$$

where, for the even modes

$$\begin{aligned} B_q &= \left(\frac{\omega_{pq}}{c\kappa_q}\right)^2 \left(\epsilon_1 \frac{\delta_q L + \sin^2(s_q L)}{\delta_q L + 1} + \epsilon_2 \frac{\cos^2(s_q L)}{\delta_q L + 1}\right), \\ a_{pq} &= -\left(\int_{-L}^{+L} u_q u_q^*\right)\left(\int_{-\infty}^{+\infty} u_q u_q^*\right)^{-1} \\ &= \frac{2\delta_q \cos^2(s_q L)}{(\delta_p + \delta_q)(1 + \delta_q L)}, \quad p \neq q \\ &= \frac{\delta_p L + \sin^2(s_p L)}{1 + \delta_p L}, \quad p = q. \end{aligned}$$

For the odd modes, the sines and cosines must be exchanged.

At the same time, the $q$th mode generates a convection current which excites the $p$th mode; therefore the coefficients $C_{q,-1}$ and $C_{p,0}$ are also related by

$$\left(-\frac{\Delta\kappa}{\kappa_p} + B_p \frac{\Delta\omega}{\omega}\right) C_{p,0} = \eta \frac{\epsilon_1}{4} \left(\frac{\omega}{c\kappa_p}\right)^2 a_{qp} C_{q,-1}, \tag{14}$$

where $B_p$ and $a_{qp}$ are equivalent to $B_q$ and $a_{pq}$ with $p$ and $q$ interchanged. Thus to satisfy Eqs. (14) and (15), we must have

$$X = (\alpha_p - \alpha_q) Y \pm [(\alpha_p + \alpha_q)^2 Y^2 - \eta^2 \gamma_{pq}^2]^{1/2}, \tag{15}$$

where

$$\begin{aligned} &X^+ = \Delta\kappa_p / K; \quad X^- = \Delta\kappa_q / K; \quad Y = \Delta\omega/\omega_{pq}; \\ &\alpha_i = (|\kappa_i|/2K) B_i \quad \text{with } i = p, q; \\ &\gamma_{pq} = \frac{\epsilon_1}{4}\left(\frac{\omega}{c}\right)^2 \frac{1}{K} \left(\frac{a_{pq} a_{qp}}{|\kappa_p \kappa_q|}\right)^{1/2}; \\ &\kappa_q = -|\kappa_q|. \end{aligned}$$

The term $X$ corresponds to the normalized change of the longitudinal wave vector when the operating frequency is equal to $\omega_{pq}(1+Y)$. The characteristic of the above solution is that the longitudinal wave vector is complex in a frequency band (called stop band) centered at $\omega_{pq}$ and of total width

$$\Omega = 2\eta\omega_{pq}\gamma_{pq}/(\alpha_p + \alpha_q) \tag{16}$$

and the imaginary component of the wave vector has a maximum equal to

$$M = \eta K \gamma_{pq}. \tag{17}$$

The corresponding Brillouin diagram is shown in Fig. 1(b). In Figs. 2 and 3 we plotted the two parameters $\Omega/\omega_{pq}$ and $1/M\lambda$ as a function of $L/\lambda$ for mode conversion (or mode coupling) from the forward basic mode to

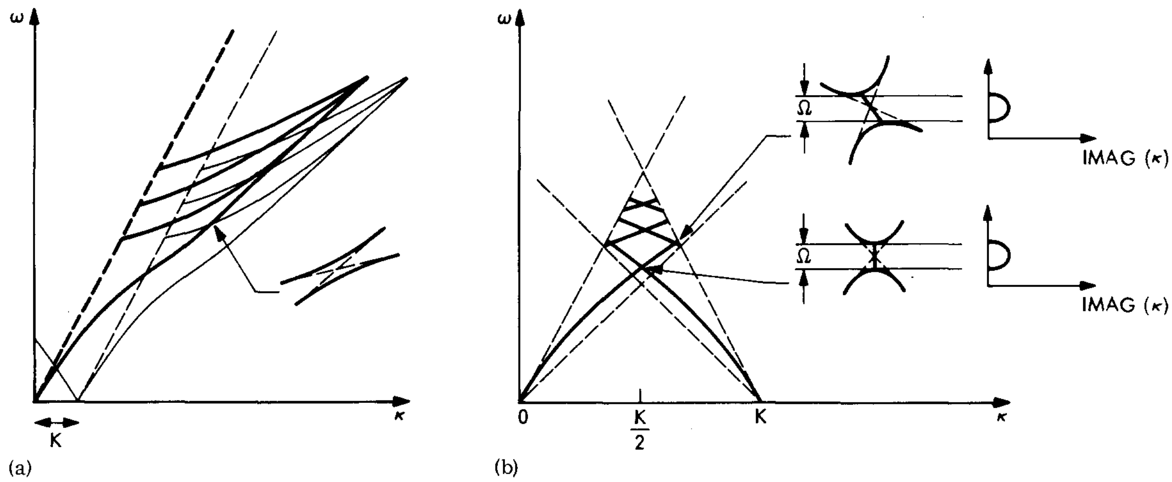



FIG. 1. Interaction regions between two space harmonics; (a) codirectional interaction, (b) contradirectional symmetric and nonsymmetric interaction.

the backward basic and second even ($p=2$) modes. $\lambda$ is the wavelength in vacuum.

As we would expect from physical reasoning, the value of $1/M\lambda$ (i.e., attenuation or coupling length) is large near cutoff because most of the energy is in the substrate where no coupling occurs. As the frequency increases, more optical energy is enclosed into the periodic guide leading to stronger coupling (i.e., $1/M\lambda$ smaller). This trend continues for the 0-0 modes coupling. However, for the 0-2 modes coupling, after a certain optimal frequency, $1/M\lambda$ starts increasing again because, at high frequencies, the overlap term $a_{pq}$ goes to zero, leading to weaker coupling. The coupling bandwidth behaves in a reverse manner. It should be pointed out that no coupling occurs between an even and an odd mode because $a_{pq}$ vanishes.

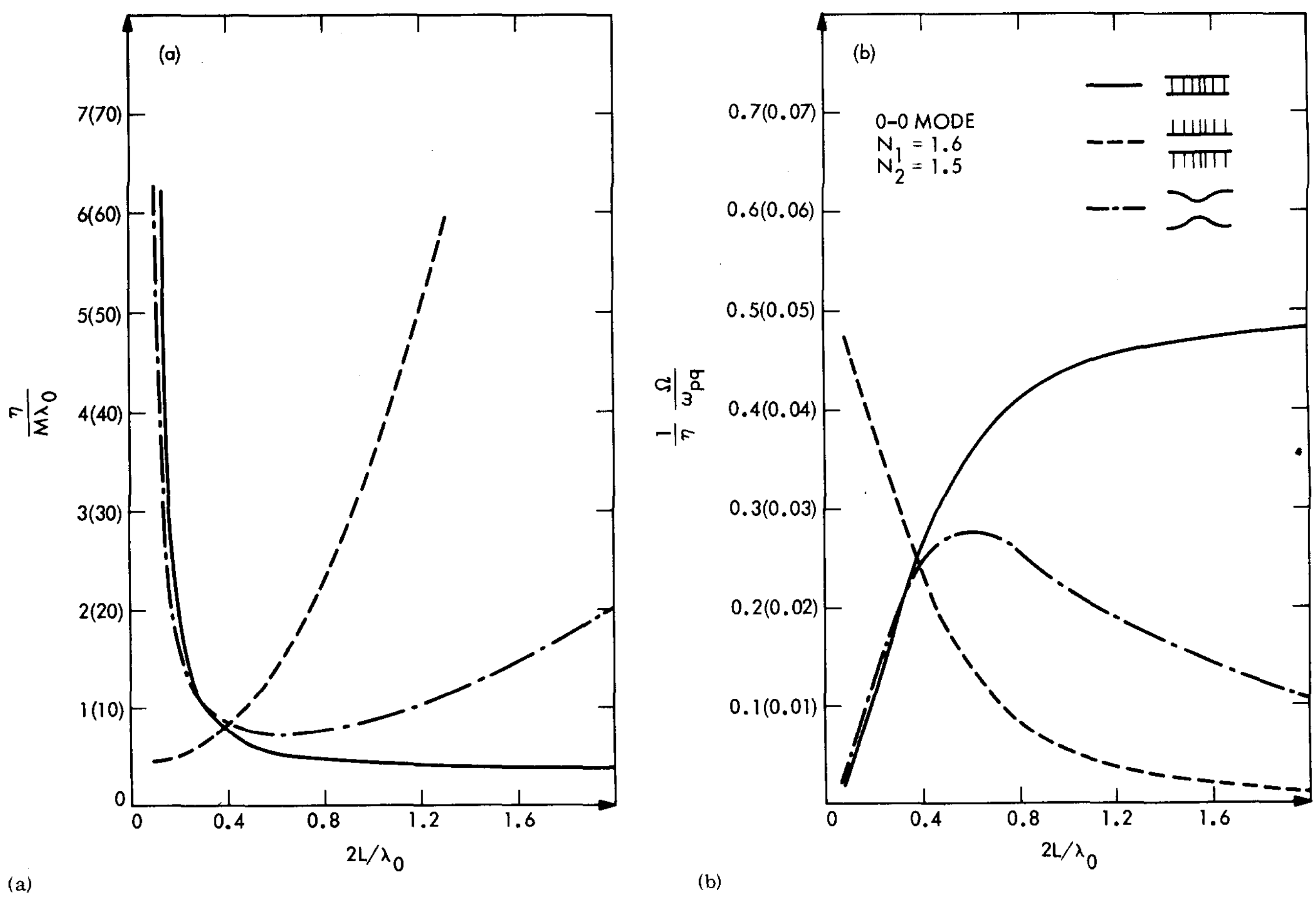
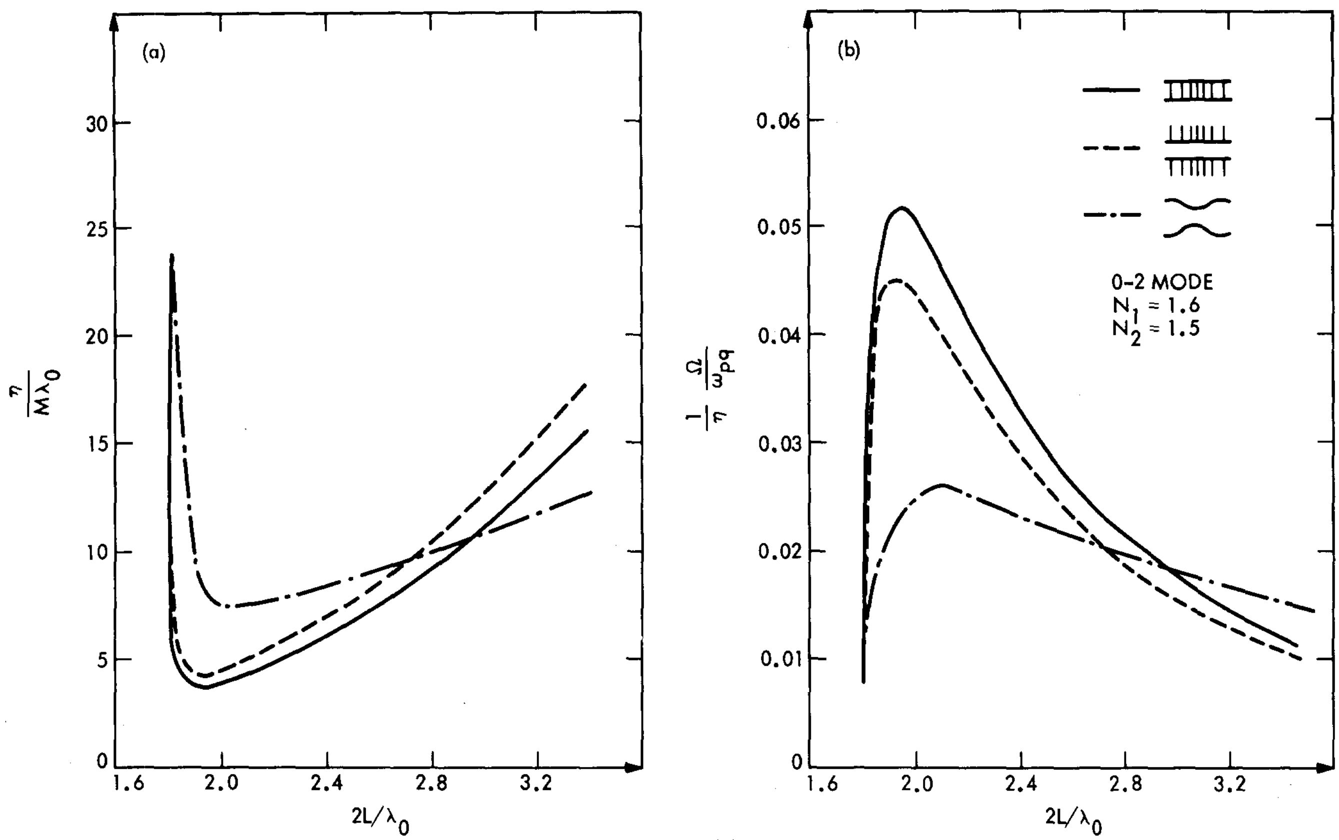



FIG. 2. Normalized interaction length and bandwidth for the 0-0 modes interaction. The scale in parenthesis corresponds to the surface corrugation case.




(a) (b)

FIG. 3. Normalized interaction length and bandwidth for the 0-2 modes interaction.

In the case of codirectional interaction, the solution for $X$ is

$$X = (\alpha_p + \alpha_q)Y \pm [(\alpha_p - \alpha_q)^2 \; Y^2 + \pi^2\gamma_{pq}^2]^{1/2} \tag{18}$$

and the corresponding diagram is shown in Fig. 3(a). In this case there is no stop band, but the energy is transferred back and forth between the two modes. The characteristic energy transfer length $T_{pq}$ is given by

$$T_{pq} = \pi/2M$$

which can be determined from Figs. 2 and 3.

It should be mentioned that, in the case $p = q$, the above results are identical to the ones derived in Ref. 4 using a direct wave equation solution in a periodic medium.

## IV. MODE CONVERSION IN A HOMOGENEOUS WAVEGUIDE WITH A PERIODIC SUBSTRATE

The same method used above can also be applied to the case where the waveguide is homogeneous and the substrate has a periodic dielectric constant. In this case, the source convection current is in the substrate, and Eqs. (16) and (19) are valid with $\gamma_{pq}$ replaced by

$$\Gamma_{pq} = \frac{\epsilon_2}{4}\left(\frac{\omega}{c}\right)^2 \frac{1}{K}\left(\frac{b_{pq}b_{qp}}{|\kappa_p\kappa_q|}\right)^{1/2}, \tag{19}$$

where

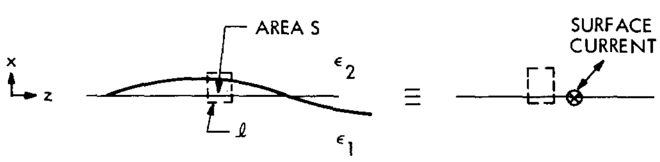
$$b_{pq} = \left(\int_{-\infty}^{-L} u_p u_q^* + \int_{L}^{+\infty} u_p u_q^*\right)\left(\int_{-\infty}^{+\infty} u_q u_q^*\right)^{-1}$$

$$= \frac{2\delta_q \cos^2(s_q L)}{(\delta_p + \delta_q)(1 + \delta_q L)}.$$

We remark that for $p \neq q$

$$\Gamma_{pq} = (\epsilon_2/\epsilon_1)\gamma_{pq}. \tag{20}$$

This is an important result because it means that for $\epsilon_2$ very close to $\epsilon_1$, the coupling between different modes does not depend appreciably on the fact that the periodicity is in the guide or the substrate. In Figs. 2 and 3 we plotted the corresponding values of $\Omega/\omega_{pq}$ and $1/M\lambda$. For the 0-0 coupling, $1/M\lambda$ increases with the frequency because the energy tends to concentrate into the guide leading to weak coupling. In the case of the 0-2 interaction, the coupling is also weak (i.e., $1/M\lambda$ large) near the mode 2 cutoff because its energy is very thinly spread into the substrate, and the overlap integral $b_{pq}$ with the 0 mode is very small.




(a) (b)

FIG. 4. A surface perturbation could be replaced by a surface current.

## V. MODE CONVERSION IN A HOMOGENEOUS GUIDE WITH A PERIODIC BOUNDARY

A similar approach can be used to study the case where the boundary between the waveguide and the substrate is periodically perturbed. Marcuse[9] determined the energy losses in such a guide by considering the boundary region as a thin inhomogeneous layer. In a previous paper[4] the authors determined the coupling between identical modes by an exact solution. Here we will determine the coupling strength and bandwidth by using an excitation surface current to represent the surface perturbation.

Let us consider the $p$th mode wave of frequency $\omega = \omega_{pq} + \Delta\omega$. From Maxwell's equations we know that (see Fig. 4)

$$\int_l \mathbf{H} = -i\epsilon_0\epsilon_1\omega \int_s \mathbf{E}. \tag{21}$$

The surface perturbation is equivalent to a surface current, $\mathbf{J}_s$, on a straight boundary, which is related to the fields by

$$\int_l \mathbf{H} = -i\epsilon_0\epsilon_2\omega \int_s \mathbf{E} + \int_s \mathbf{J}_s. \tag{22}$$

The above two equations imply that

$$\int_s \mathbf{J}_s = -i\epsilon_0(\epsilon_1 - \epsilon_2)\omega \int_s \mathbf{E} \Rightarrow$$

$$J_s \Delta z = -i\epsilon_0(\epsilon_1 - \epsilon_2)\omega \Delta x \Delta z E\big|_{x=L}$$

which gives the value of $J_s$ [using $\Delta x = \eta L \cos(Kz)$]

$$J_s = -i\epsilon_0 \eta L(\epsilon_1 - \epsilon_2)\omega \cos(Kz) C_{p0} \exp(i\kappa_p' z). \tag{23}$$

This surface current could be phase matched with the neighboring space harmonics. For the case of contradirectional phase matching [Eq. (8)] a backward $q$th mode is excited. The boundary condition for this new mode is

$$H_z(x = L^+) - H_z(x = L^-) = J_{s1}, \tag{24}$$

where $J_{s1}$ is the component in phase with the generated wave. The above equation gives

$$[s_q' \tan(s_q' L) - \delta_q'] C_{q,-1} = \tfrac{1}{2}\eta(\epsilon_1 - \epsilon_2) L(\omega_{pq}/c) C_{p0} \tag{25}$$

for the even modes. For the odd modes, the tangent must be replaced by minus cotangent. At the same time, the $q$th mode excites the $p$th mode, leading a relation similar to Eq. (25) (with $p$ and $q$ exchanged), which then implies that

$$[s_q' \tan(s_q' L) - \delta_q'][s_p' \tan(s_p' L) - \delta_p'] = [\tfrac{1}{2}\eta(\epsilon_1 - \epsilon_2)L]^2(\omega_{pq}/c)^4. \tag{26}$$

This relation replaces the boundary condition [Eq. (2c)] of the homogeneous guide, and couples the two modes through the perturbation of the boundary. The other dispersion relations [Eqs. (2a) and (2b)] are still valid.

Replacing $s'$, $\delta'$, and $\kappa'$ by their expressions and following the same method as previously we find that Eq. (16) is still valid with $\gamma_{pq}$ replaced by

$$\gamma_{pq}' = \frac{\epsilon_1 - \epsilon_2}{2}\left(\frac{\omega}{c}\right)^2 \frac{1}{K}\left(\frac{d_p d_q}{|\kappa_p \kappa_q|}\right)^{1/2}, \tag{27}$$

where

$$d_i = \frac{\delta_i L \cos^2(s_i L)}{1 + \delta_i L}, \quad i = p, q.$$

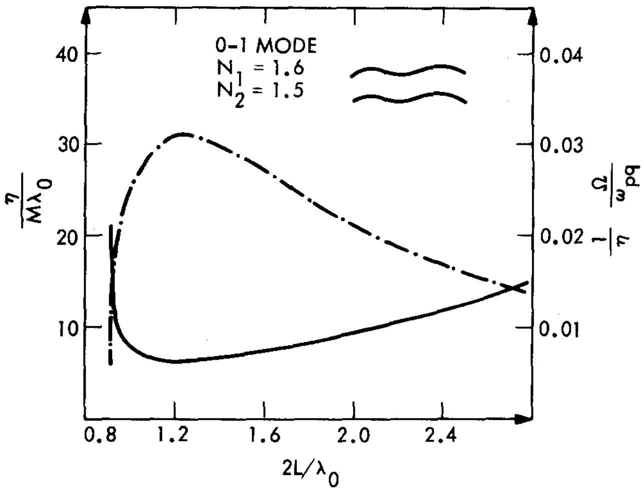



FIG. 5. Normalized interaction length and bandwidth for the 0-1 modes interaction in an antisymmetrically corrugated guide.

For an odd mode, the cosine should be replaced by sine. This result is identical to the one derived by the authors[4] using the exact Floquet solution.

In a symmetrically perturbed waveguide only even-even and odd-odd couplings occur. Even-odd couplings occur in the antisymmetric case. If only one boundary is perturbed, then the above results are still valid with $\eta$ replaced by $\frac{1}{2}\eta$.

In Figs. 2, 3, and 5 we plotted $\eta/M\lambda$ and $\Omega/\eta\omega_{pq}$ for the 0-0, 0-1, and 0-2 modes coupling. We see that near cutoff, where most of the energy is spread in the substrate, the coupling is weak. The same happens at high frequency when most of the energy is inside the guide and the field is small at the boundary. The strongest coupling (and therefore feefback or filtering) occurs at an optimum frequency somewhere inbetween.

## VI. APPLICATIONS AND CONCLUSION

The results of this paper can be applied to the design of integrated optic filters and distributed feedback lasers. The terms $\eta/M_{pq}$ and $\Omega_{pq}$ describe the coupling, filtering, and feedback efficiency. The coupling coefficient can be used to determine the gain threshold of a thin-film distributed feedback laser.[11] It is clear that for an optimum design the operating frequency has to be selected in a well-specified region. It should be pointed out that the product

$$\frac{\Omega}{\omega_{pq}} \frac{1}{M\lambda} = \frac{2}{\lambda(\alpha_p + \alpha_q)} \tag{28}$$

is the same for all three structures studied and depends only on the properties of the unperturbed guide and the operating frequency. This implies that if a structure is designed for a large coupling coefficient, then its bandwidth would be very narrow and vice versa.

Even though this paper was limited to the case of thin-film waveguides, the same methods could be used to study other types of structures: diffuse waveguides for DFB lasers and filters, diffuse capillary waveguides for $CO_2$ lasers, and fiber waveguides.

Finally, it should be mentioned that the above results could be easily generalized to any type of periodicity which can be expanded in a Fourier series.[12] In that case, $\eta$ has to be replaced by the coefficient of the Fourier component used for phase matching.

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